



Advantages of an Easy to Design Fuzzy Predictive Algorithm: Application to a Nonlinear Chemical Reactor

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Abstract. Advantages of a fuzzy predictive control algorithm are discussed in the paper. The discussed fuzzy predictive algorithm is a combination of a DMC (Dynamic Matrix Control) algorithm and Takagi–Sugeno fuzzy modeling inheriting advantages of both techniques. The algorithm is numerically effective. Moreover, information about measured disturbance can be included in it in an easy way. A simple and easy to apply method of fuzzy predictive control algorithms synthesis is presented. The advantages of the fuzzy predictive control algorithm are demonstrated in the example control system of a control plant with difficult dynamics – a nonlinear chemical reactor with inverse response.

1 Introduction

Model predictive control (MPC) algorithms are widely used in practice due to the control performance they offer. They are usually used in an advanced control layer of the multilayer control system structure; see e.g. [1, 8, 10]. Basic formulations of the MPC algorithms are based on the linear control plant models. The advantage of these algorithms is that they are formulated as easy to solve, linear–quadratic optimization problems. However, application of the MPC algorithm based on a linear plant model to a nonlinear plant, if control in a wide range of set point values is needed, may bring unsatisfactory results or the results can be improved using the algorithm based on a nonlinear model.

Unfortunately, a direct usage of a nonlinear model to design the MPC algorithm leads to its formulation as a nonlinear, and in general, non–convex optimization problem that is hard to solve and computationally demanding. Thus, in practice, suboptimal algorithms that use approximate process models generated from the nonlinear model at each algorithm iteration are used; see e.g. [6, 10]. In these algorithms the nonlinear and the approximated models are used in such a way that the standard linear–quadratic optimization problem is formulated and solved at each algorithm iteration. Various kinds of nonlinear models can be used in such algorithms. However, the usage of Takagi–Sugeno (TS) fuzzy models [9] makes them especially efficient [6, 7, 10].

The DMC algorithm is practically a standard in the industrial applications; see e.g. [1, 2, 3, 5, 8, 10]. It can be relatively easily designed, because it is based on an easy to

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obtain control plant model in the form of the step response. The fuzzy DMC (FDMC) algorithm used in the paper is an easy to design fuzzy predictive algorithm based on DMC technique and Takagi–Sugeno fuzzy models [6, 7, 10]. It uses a TS fuzzy model with step responses used as the local models. Such Takagi–Sugeno fuzzy models can be obtained relatively easy using the expert knowledge and/or simulation experiments.

As the FDMC algorithm is based on the nonlinear control plant model, it usually works well in a wide range of operating point changes, offering better control performance than the algorithms based on linear process models – the feature important when it operates in a hierarchical control system structure.

The discussed FDMC algorithm was applied to a nonlinear chemical plant – an isothermal CSTR in which a van de Vusse reaction takes place. The control plant is a difficult process because except it is nonlinear, it has also an inverse response. The experiments performed in two control systems: the first one – with DMC algorithm based on a linear process model and the second one – with FDMC algorithm, demonstrate the superiority of the fuzzy algorithm over the one based on the linear model.

The organization of the paper is as follows. In Sect. 2 the idea of predictive control algorithms is discussed, the DMC and easy to design FDMC algorithms used during the test are described. In Sect. 3 the control plant and algorithms used during the experiments are detailed. Sect. 4 contains description of experiments and discussion of obtained results. The paper is summarized in Sect. 5.

2. Predictive Controllers

In the MPC algorithms future behavior of the control plant is predicted many time steps ahead, using a dynamic control plant model and all available knowledge about conditions of control system operation. The future control values are then derived in such a way that the future behavior of the control system fulfills assumed criteria. Typically, the minimization of a performance function is demanded subject to the constraints put on manipulated and controlled variables; in the MPC algorithms based on input–output models usually the following optimization problem is solved at each iteration:

$$\min_{\Delta \mathbf{u}} \left\{ J_{MPC} = \sum_{i=1}^p (\bar{y}_k - y_{k+i|k})^2 + \sum_{i=0}^{s-1} \lambda \cdot (\Delta u_{k+i|k})^2 \right\} \quad (1)$$

$$\text{subject to: } \Delta \mathbf{u}_{\min} \leq \Delta \mathbf{u} \leq \Delta \mathbf{u}_{\max}, \mathbf{u}_{\min} \leq \mathbf{u} \leq \mathbf{u}_{\max}, \mathbf{y}_{\min} \leq \mathbf{y} \leq \mathbf{y}_{\max},$$

where \bar{y}_k is a set–point value (in the classical multilayer control system structure it is obtained from the economic optimization layer), $y_{k+i|k}$ is an output value for $(k+i)^{\text{th}}$ time step predicted at k^{th} time step using control plant model, it depends on past and future control values, $\Delta u_{k+i|k}$ are future changes in the manipulated variables, $\lambda \geq 0$ is a weighting coefficient, p and s denote prediction and control

horizons, respectively; $y = [y_{k+1|k}, \dots, y_{k+p|k}]^T$, $\Delta u = [\Delta u_{k|k}, \dots, \Delta u_{k+s-1|k}]^T$, $u = [u_{k|k}, \dots, u_{k+s-1|k}]^T$, Δu_{min} , Δu_{max} , u_{min} , u_{max} , y_{min} , y_{max} are vectors of lower and upper bounds of changes and values of the control signals and of the output variable values, respectively. As a solution to the optimization problem (1) the optimal vector of changes of the manipulated variables is obtained. From this vector, the $\Delta u_{k|k}$ element is taken and applied in the control system. Then optimization is repeated at the next time instant.

The way the predicted output values $y_{k+i|k}$ are derived depends on the dynamic control plant model exploited by the algorithm. If the linear model is used then the optimization problem (1) is a standard linear–quadratic programming problem. The conventional (non–fuzzy) DMC algorithm is based on a process model in the form of the control plant step responses.

2.1. Standard DMC Algorithm

The classical (non–fuzzy) DMC algorithm uses the control plant model in the form of its step response; see e.g. [2, 3, 5, 8, 10]:

$$\tilde{y}_k = \sum_{i=1}^{p_d-1} a_i \cdot \Delta u_{k-i} + a_{p_d} \cdot u_{k-p_d}, \quad (2)$$

where \tilde{y}_k is the output of the control plant model at the k^{th} time instant, Δu_k is a change of the manipulated variable at the k^{th} time instant, a_i ($i=1, \dots, p_d$) are step response coefficients of the control plant, p_d is equal to the number of time instants after which the coefficients of the step response can be assumed as settled, u_{k-p_d} is a value of the manipulated variable at the $(k-p_d)^{\text{th}}$ time instant.

The predicted output values are then calculated using the following formula:

$$y_{k+i|k} = \sum_{n=1}^i a_n \cdot \Delta u_{k-n+i} + \sum_{n=i+1}^{p_d-1} a_n \cdot \Delta u_{k-n+i} + a_{p_d} \cdot u_{k-p_d+i} + d_k, \quad (3)$$

where $d_k = y_k - \tilde{y}_{k-1}$ is assumed to be the same at each time instant in the prediction horizon (it is a DMC–type disturbance model). Thus (3) can be transformed into the following form:

$$y_{k+i|k} = y_k + \sum_{n=i+1}^{p_d-1} a_n \cdot \Delta u_{k-n+i} + a_{p_d} \sum_{n=p_d}^{p_d+i-1} \Delta u_{k-n+i} - \sum_{n=1}^{p_d-1} a_n \Delta u_{k-n} + \sum_{n=1}^i a_n \Delta u_{k-n+i|k}. \quad (4)$$

In (4) only the last component depends on future manipulated variable changes. Thus the vector of predicted output values \mathbf{y} can be decomposed into the following components:

$$\mathbf{y} = \mathbf{y}^{fr} + \mathbf{A} \cdot \Delta \mathbf{u}, \quad (5)$$

where $\mathbf{y}^{fr} = [\mathbf{y}_{k+1|k}^{fr}, \dots, \mathbf{y}_{k+p|k}^{fr}]^T$, is a vector of length p and is called a free response of the plant, because it contains future output values calculated assuming that the control signal does not change in the prediction horizon (describes influence of the manipulated variable values applied to the control plant in previous iterations):

$$\mathbf{y}^{fr} = \mathbf{y}^l + \mathbf{A}_p \cdot \Delta \mathbf{u}_p, \quad (6)$$

where $\Delta \mathbf{u}_p = [\Delta u_{k-1}, \dots, \Delta u_{k-p_d+1}]^T$, $\mathbf{y}^l = [y_k, \dots, y_k]^T$ and

$$\mathbf{A}_p = \begin{bmatrix} a_2 - a_1 & a_3 - a_2 & \cdots & a_{p_d-1} - a_{p_d-2} & a_{p_d} - a_{p_d-1} \\ a_3 - a_1 & a_4 - a_2 & \cdots & a_{p_d} - a_{p_d-2} & a_{p_d} - a_{p_d-1} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{p+1} - a_1 & a_{p+2} - a_2 & \cdots & a_{p_d} - a_{p_d-2} & a_{p_d} - a_{p_d-1} \end{bmatrix}. \quad (7)$$

\mathbf{A} is a matrix of dimensionality $p \times s$, called the dynamic matrix and is composed of the control plant step response coefficients:

$$\mathbf{A} = \begin{bmatrix} a_1 & 0 & \cdots & 0 & 0 \\ a_2 & a_1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ a_p & a_{p-1} & \cdots & a_{p-s+2} & a_{p-s+1} \end{bmatrix}. \quad (8)$$

Using the prediction (5) one can formulate the optimization problem (1), solution of which contains the future changes in the manipulated variables. The details concerning formulation of the DMC algorithm can be found, e.g. in [2, 3, 5, 8, 10].

Disturbance measurement can be taken into consideration in an easy way. Despite that, it can offer significant control system performance improvement. In order to take the disturbance measurement into consideration in the DMC algorithm the control plant model (2) should be supplemented by terms describing influence of the disturbance on the output variable:

$$\tilde{y}_k = \sum_{i=1}^{p_d-1} a_i \cdot \Delta u_{k-i} + a_{p_d} \cdot u_{k-p_d} + \sum_{i=1}^{p_z-1} b_i \cdot \Delta z_{k-i} + b_{p_z} \cdot z_{k-p_z}, \quad (9)$$

where Δz_k is a change of the manipulated variable at the k^{th} time instant, b_i ($i=1, \dots, p_z$) are disturbance step response coefficients, p_z is equal to the number of time instants after which the coefficients of the disturbance step response can be assumed as settled, z_{k-p_d} is a value of the manipulated variable at the $(k-p_z)^{\text{th}}$ time instant.

If there is no information available about future disturbance, the future changes of disturbance are assumed equal to 0. The influence of past changes of the disturbance should be included in the free response. Thus, this time, the free response is described by the following formula:

$$y^{\text{fr}} = y^l + A_p \cdot \Delta u_p + A_z \cdot \Delta z_p, \tag{10}$$

where $\Delta z_p = [\Delta z_{k-1}, \dots, \Delta z_{k-p_z+1}]^T$ and

$$A_z = \begin{bmatrix} a_2 - a_1 & a_3 - a_2 & \cdots & a_{p_z-1} - a_{p_z-2} & a_{p_z} - a_{p_z-1} \\ a_3 - a_1 & a_4 - a_2 & \cdots & a_{p_z} - a_{p_z-2} & a_{p_z} - a_{p_z-1} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{p+1} - a_1 & a_{p+2} - a_2 & \cdots & a_{p_z} - a_{p_z-2} & a_{p_z} - a_{p_z-1} \end{bmatrix}. \tag{11}$$

2.2. Easy to Design Fuzzy DMC Algorithm

The FDMC algorithm used in the paper exploits TS models with local models in the form of step responses. Thanks to such an approach each iteration of the algorithm is simple. Moreover, the local models are easy to obtain. The FDMC algorithm is thus based on TS fuzzy models described by the set of the following rules:

Rule j : if

$$\underbrace{y_k \text{ is } B_l^j \text{ and } \dots \text{ and } y_{k-n_p+1} \text{ is } B_{n_p}^j \text{ and } u_k \text{ is } C_l^j \text{ and } \dots \text{ and } u_{k-m_p+1} \text{ is } C_{m_p}^j}_{\text{antecedent}} \tag{12}$$

$$\text{then } \underbrace{\tilde{y}_{k+l}^j = \sum_{i=1}^{p_d-1} a_i^j \cdot \Delta u_{k-i} + a_{p_d}^j u_{k-p_d}}_{\text{consequent}},$$

where y_k is an output variable value of the control plant model at the k^{th} time instant, u_k is a manipulated variable value at the k^{th} time instant, $B_1^j, \dots, B_{n_p}^j, C_1^j, \dots, C_{m_p}^j$ are fuzzy sets, a_i^j are the coefficients of step responses in j^{th} local model, $j=1, \dots, l$, l – number of rules.

If measured disturbance is taken into consideration, the local model has the following form:

$$\tilde{y}_{k+l}^j = \sum_{i=1}^{p_d-1} a_i^j \cdot \Delta u_{k-i} + a_{p_d}^j \cdot u_{k-p_d} + \sum_{i=1}^{p_z-1} b_i^j \cdot \Delta z_{k-i} + b_{p_z}^j \cdot z_{k-p_z}, \quad (13)$$

where b_i^j are the coefficients of step responses in j^{th} local model.

The design process of the TS model (12) can be simplified to a large extent. It is because instead of obtaining a complicated local models (as e.g. difference equations) it is sufficient to obtain a few step responses of the control plant (from the environs of a few operating points). The membership functions can be chosen using expert knowledge, simulation experiments, fuzzy neural networks or all these techniques combined.

In the basic FDMC algorithm, the TS model (12) is used, at each iteration, in the following way:

1. A linear model for current time instant is derived using current values of process variables, the TS fuzzy model (12) and fuzzy reasoning. The output value of the model is calculated using the following formula (in the case when a measured disturbance is taken into consideration):

$$\tilde{y}_k = \sum_{i=1}^{p_d-1} \tilde{a}_i \cdot \Delta u_{k-i} + \tilde{a}_{p_d} \cdot u_{k-p_d} + \sum_{i=1}^{p_z-1} \tilde{b}_i \cdot \Delta z_{k-i} + \tilde{b}_{p_z} \cdot z_{k-p_z}. \quad (14)$$

where $\tilde{a}_i = \sum_{j=1}^l \tilde{w}_j \cdot a_i^j$, $\tilde{b}_i = \sum_{j=1}^l \tilde{w}_j \cdot b_i^j$ and \tilde{w}_i are the normalized weights calculated using standard fuzzy reasoning, see e.g. [9].

In fact (14) is the step response control plant model valid for the current values of process variables. Thus, next steps of the FDMC algorithm are the same as in the conventional DMC algorithm (Sect. 2.1), i.e.

2. The obtained step response coefficients are used to generate the dynamic matrix.
3. The step response model (14) is used to generate the plant free response.
4. The free response and the dynamic matrix are used to formulate the quadratic optimization problem.
5. The optimization problem is solved and, using the obtained solution, the manipulated variable value is generated. Then the controller passes to the next iteration.

3. Control systems

The considered control plant is an isothermal CSTR in which a van de Vusse reaction carries out (Fig. 1 a) [4]. Its steady-state characteristics are shown in Fig. 1 b.

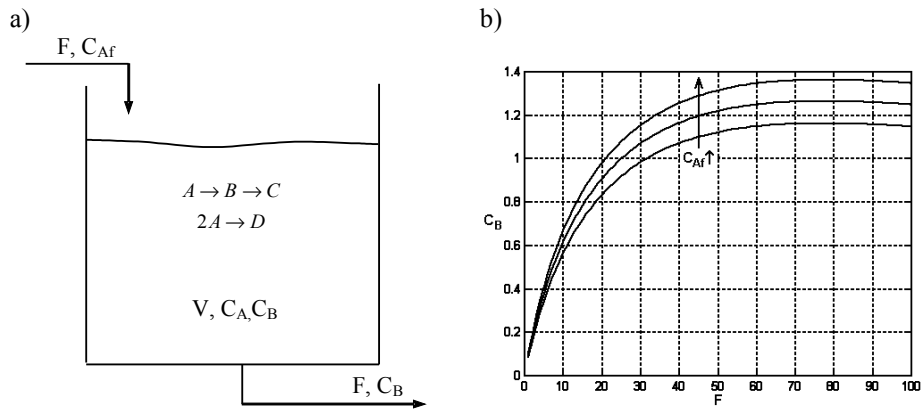
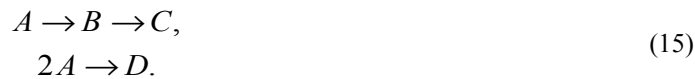


Fig. 1. Isothermal CSTR with van de Vusse reaction; a) diagram of the plant, b) steady-state characteristics of the plant

The reaction scheme is as follows



The process model contains composition balance equations for both components

$$\begin{aligned} \frac{dC_A}{dt} &= -k_1 C_A - k_3 C_A^2 + \frac{F}{V} (C_{Af} - C_A), \\ \frac{dC_B}{dt} &= k_1 C_A - k_2 C_B - \frac{F}{V} C_B. \end{aligned} \quad (16)$$

where C_A , C_B are the concentrations of components A and B, respectively, F is the inlet flow rate (equal to the outlet flow rate), V is the volume in which the reaction is carried out (it is assumed constant and $V=1$ l), C_{Af} is the concentration of component A in the inlet flow stream (if it is not stated differently it is assumed that $C_{Af0}=10$ mol/l). The values of the kinetic parameters are: $k_1=50$ 1/h, $k_2=100$ 1/h, $k_3=10$ 1/h · mol.

The output variable is the concentration C_B of substance B, the manipulated variable is the inlet flow rate F of the raw substance, the disturbance variable is the concentration of component A in the inlet flow stream C_{Af} .

It was assumed that the manipulated variable is constrained

$$F_{\min} \leq F \leq F_{\max} \quad (17)$$

where $F_{\min}=0$ l/h, $F_{\max}=60$ l/h.

For the presented control plant two controllers were designed: the first one – a DMC controller based on a linear model and the second one – an FDMC controller based on a Takagi–Sugeno fuzzy model. The sampling period of both controllers was assumed equal to 3.6 s. The following tuning parameters of both algorithms were assumed: $p=70$, $s=35$, $\lambda=0.001$.

The DMC algorithm is based on a step response taken in environs of the operating point: $C_{B0} = 1.12$ mol/l, $C_{A0} = 3$ mol/l, $F = 34.3$ l/h [4]. In the case of an FDMC algorithm, two additional step responses are used (the conventional DMC algorithm was in fact extended): the first one from environs of the point $C_{B0} = 0.91$ mol/l, $C_{A0} = 2.18$ mol/l, $F = 20$ l/h, and the second one – from environs of the point $C_{B0} = 1.22$ mol/l, $C_{A0} = 3.66$ mol/l, $F = 50$ l/h. Thus, the TS control plant model used by the FDMC controller is composed of three rules. The assumed membership functions are shown in Fig. 2.

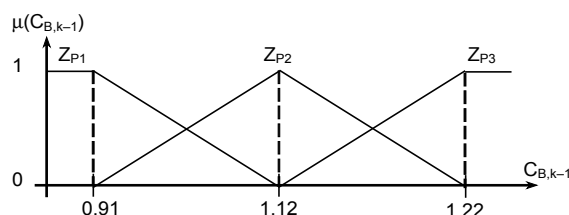


Fig. 2. Membership functions of the fuzzy DMC controller

4. Simulation Experiments

During the experiments the operation of both control systems (the first one – with DMC and the second one – with FDMC algorithms) was compared. First, the set-point value \bar{C}_B was changed towards high values of the concentration C_B , to 1.22 mol/l, precisely. The response obtained in the control system with the FDMC algorithm (solid lines in Fig. 3) is better than the one acquired by the conventional DMC algorithm (dashed lines in Fig. 3) – the output variable much faster reaches the demanded set-point value.

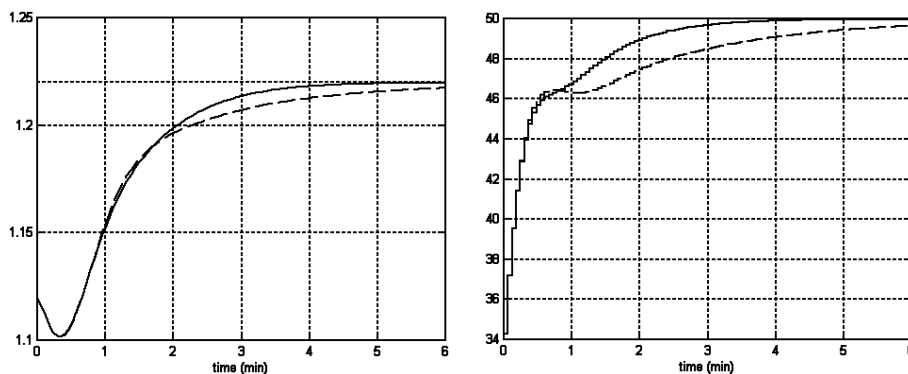


Fig. 3. Responses of the control system with: DMC algorithm (dashed line), FDMC algorithm (solid line) to the change of C_B set-point to $\bar{C}_B = 1.22$; left – output variable C_B , right – manipulated variable F

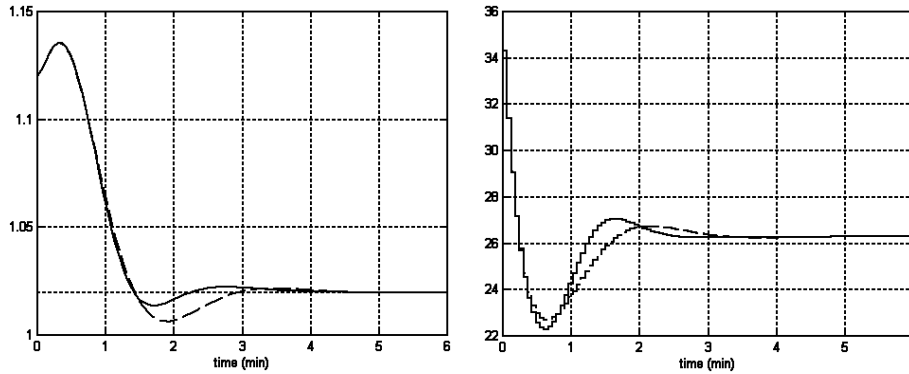


Fig. 4. Responses of the control system with: DMC algorithm (dashed line), FDMC algorithm (solid line) to the change of C_B set-point to $\bar{C}_B = 1.02$; left – output variable C_B , right – manipulated variable F

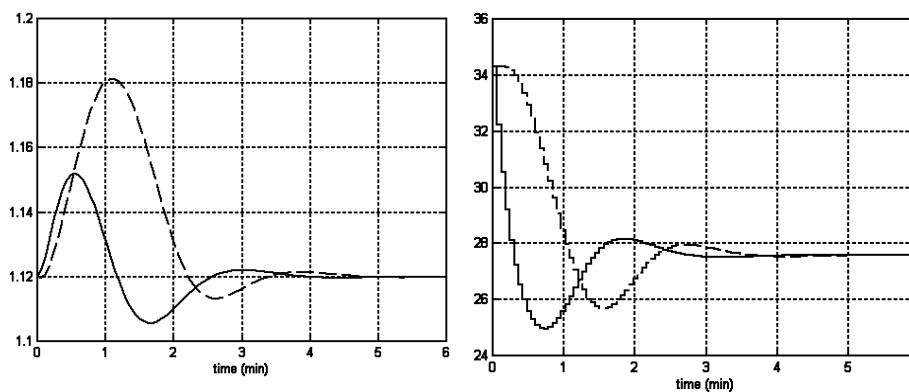


Fig. 5. Responses of the control system with FDMC algorithm to the increase of disturbance C_{Af} by 1 mol/l; disturbance measurement mechanism: not used (dashed line), used (solid line) in the algorithm; left – output variable C_B , right – manipulated variable F

It can be also seen that both responses at the beginning of control action (for C_B values close to the 1.12 mol/l) are practically the same. It is caused by the fact that the responses, the conventional DMC controller is based on, were used as a local model in one of the rules of the TS model exploited by the FDMC controller. This fact illustrates well that the FDMC controller can be designed by simply extending the existing DMC controller (usually well tuned to work near the operating point it was designed for) if needed.

In the second experiment, the set-point value \bar{C}_B was changed towards low values of the concentration C_B , i.e. to 1.02 mol/l. As in the previous experiment, the response obtained in the control system with FDMC algorithm (solid lines in Fig. 4) is better than the one generated by the control system with the conventional DMC algorithm (dashed lines in Fig. 4) – the output variable also reaches the set-point

value faster in the case of the control system with the fuzzy algorithm. Moreover, the overshoot is significantly lower in the control system with the FDMC algorithm. The obtained results clearly show the profits gained thanks to the usage of the fuzzy approach.

There was also performed an experiment with the change of disturbance C_{Af} – concentration of component A in the inlet stream. In the case when there was a disturbance measurement used (solid lines in Fig. 5) the obtained settling time is smaller than in the case without disturbance measurement (dashed lines in Fig. 5). Moreover, in the first case there is much smaller overshoot. Similar results were obtained in the case of operation near other operating points. In such cases the further the original operating point the system operates, the bigger difference of operation between DMC and FDMC algorithm is, with advantage of the latter one.

5. Summary

Application of the FDMC algorithm to the example control plant brought improvement of the control system operation comparing to the case when the algorithm based on the linear model was used. It was achieved despite relatively little effort needed to design a fuzzy algorithm. It was possible because model the FDMC algorithm is based on (a Takagi–Sugeno fuzzy model with step responses used as local models) can be obtained in an easy way. The mechanism of taking measured disturbance into consideration in the algorithm (also relatively simple to apply), brought further improvement of algorithm performance.

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