



Applications of Finite Element Method in Synovial Joint Numerical Calculations

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Abstract. The problem of meshing for numerical calculations of human rapid movement influence on the tribological features of a synovial joint was discussed. The calculations performed pointed out great effectiveness of the ADINA-F solver.

1. Introduction

Biomechanics aims at describing human movement under dynamic conditions. The hip joint is one of the elements of the human osteoarticular system that are the most exposed to overload-related degenerative changes. The use of computer methods to solve equations which model the phenomena taking place in the human body is getting more and more common and brings about better and better results. In the case of research into the hip joint biobearing, the methods provide the information which may be useful for medical sciences and endoprosthesis design.

The aim of the research whose results are presented in the paper was to explain how human effort movements affect the tribological properties of the biobearing. In a previous work we presented highly accurate finite element method calculations of synovial joint tribology using the ADINA-F package. To dynamically describe the synovial fluid behavior, the nonstationary Navier Stokes equations were used. The calculations were done making use of the Galerkin weighted residual method, the Newton-Raphson method and the Gauss elimination method. To perform the calculations tetrahedral mesh elements were applied.

In this paper we focus on problems with optimal meshing in the numerical hip flow simulation. Numerical calculations of tribological values of the biobearing during effort movements were carried out. They were modeled dynamically, i.e. the results

were obtained by solving nonstationary equations of synovial fluid flow in the hip joint by the finite elements method making use of an ADINA-F software package. The problem was solved for a simplified model of a hip joint, ignoring the properties of articular cartilage – its permeability and elasticity.

2. Problem formulation

The assumed geometry of the articular fissure was based on a normal joint measurements. The situations considered were those encountered during getting up or jumping, which was modeled by moving the articular surfaces to each other.

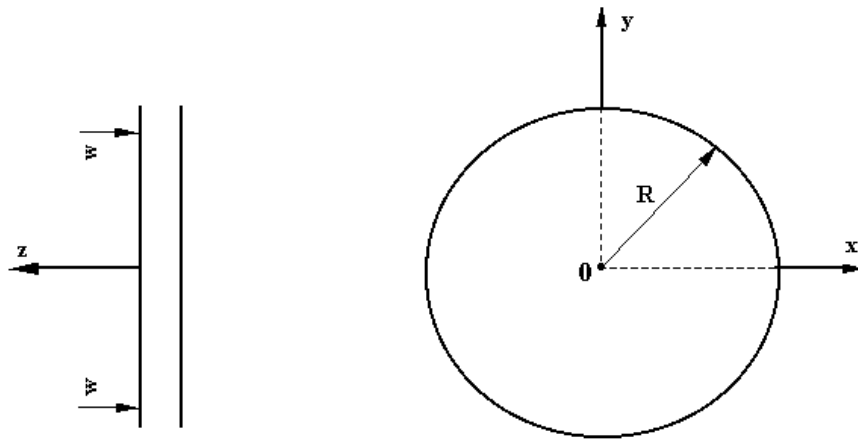


Fig1. Geometry of the flow.

The solution of the flow problem was obtained for a very much simplified geometrical model of the joint

- the flow area in the articular fissure was modeled as an area between parallel, circular plates,
- it was assumed that the surface of the plates is hard and impermeable (for the time being, elasticity and porosity of cartilage were neglected),
- initial (h_p) and final (h_k) heights of the lubrication fissure were assumed, supposing at the same time that the plate will displace from the initial to the final position in time t ,
- mobile plate moves only along axis z

- as our purpose was to obtain qualitative results, the Newtonian model of synovial fluid was tentatively adopted.

The geometry of the flow is presented on Fig.1. Synovial flow in a channel was described by the Navier – Stokes system of equations, which, together with the equation of flow continuity for the analyzed problem is as follows:

$$\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = - \frac{\partial p}{\partial x} + \eta \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \quad (2.1)$$

$$\rho \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = - \frac{\partial p}{\partial y} + \eta \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \quad (2.2)$$

$$\rho \left(\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) = - \frac{\partial p}{\partial z} + \eta \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) \quad (2.3)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (2.4)$$

The boundary conditions are:

$$\mathbf{u}=0, \mathbf{v}=0, \mathbf{w}=0 \text{ on the surface of the stationary plate,} \quad (2.5)$$

$$\mathbf{u}=0, \mathbf{v}=0, \text{ on the surface of the movable plate,} \quad (2.6)$$

$$\mathbf{p}=0 \text{ on the edges of the stationary and movable plates,} \quad (2.7)$$

t -time of plate relocation from the initial to final position

$$(2.8)$$

where: $\mathbf{V}=(\mathbf{u}, \mathbf{v}, \mathbf{w})$ -synovial fluid velocity , \mathbf{p} – pressure in the fluid layer [Pa]

The following value of parameters – resulting from the physiology of the joint – were used for the calculations:

h_p – initial height of the articular fissure, $h_p = 200 \mu\text{m}$,

h_k – final height of the articular fissure, $h_k = 40 \mu\text{m}$

t – time of the plate displacement from the initial to final position; six values of the parameter were used for the calculations: $t_1 = 0,0005s$, $t_2 = 0,001s$, $t_3 = 0,003s$, $t_4 = 0,005s$, $t_5 = 0,0075s$, $t_6 = 0,01s$,

R – radius of the plate, $R=0.01m$

η – dynamic viscosity of the synovial fluid, $\eta=0,116 Pa\cdot s$,

ρ – synovial fluid density; $\rho = 1000 kg/m^3$.

3. Methodology of the calculations

The finite elements method and ADINA 8.1 software packages were applied to solve the (1)-(4) equations with (5)-(8) boundary conditions. The calculations were done making use of the Galerkin weighted residual method, the Newton-Raphson method and the Gauss elimination method. Tetrahedral-tetranodal elements were chosen to calculate the flow in the bearing.

The number of elements which is required for a given accuracy depends on the flow geometry. The mesh in ADINA can be generated either automatically or by the user. For the calculations the mesh thickened in the place of maximum pressure of the bioflow was created. Generated and tested and were four variants, each with a different number of elements.

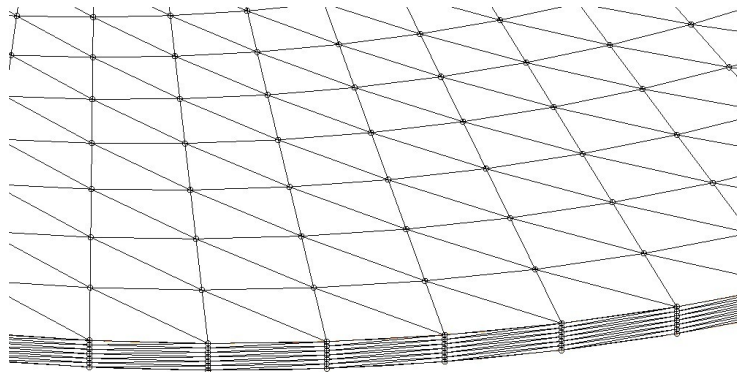


Fig2. Mesh plot.

The calculated values of physical quantities were accepted as a criterion for comparing the four variants with the analytical solutions They were:

- maximum pressure in the synovial film,

- maximum flow velocity values for synovia u_{\max} , v_{\max}

The results of the calculations are presented in the Table 1.

Table 1 Effect of Mesh density on calculations

Notation	Number of nodes	Number of elements	pmax [Pa]	u_{\max} [m/s]	v_{\max} [m/s]
S1	4448	19440	2,197·10 ⁵	1,468	1,468
S2	8995	42240	2,099·10 ⁵	1,580	1,580
S3	19803	100080	2,034·10 ⁵	1,534	1,534
S4	28126	142560	2,034·10 ⁵	1,534	1,534

An analysis of the results pointed out that the variant marked as S3 (19803 nodes and 100080 elements) ensured the satisfactory accuracy of the calculation results.

4. Results

Numerical calculations of tribological values of the biobearing during effort movements were carried out. They were modeled dynamically, by six different times of the approaching plates. The results are presented in the Table 2. The obtained values of the calculations of the joint load are from the range of real values [1, 2] and show qualitative agreement with the calculations based on the micropolar model of synovial fluid [3].

Table 2. Calculations Results

approach time of the plates, t [s]	load capacity W [kN]	p_{sr} [MPa]	p_{\max} [MPa]	u_{\max} [m/]	v_{\max} [m/]	W_{\max} [m/s]
0,0005	28,5	90,43	181,29	61,33	61,33	0,325
0,0010	14,1	44,92	89,93	30,72	30,72	0,166
0,0030	4,68	14,85	29,80	10,23	10,23	0,055
0,0050	2,81	8,90	17,95	6,14	6,14	0,033
0,0075	1,87	5,93	11,89	4,09	4,09	0,022
0,0100	1,41	4,45	8,92	3,07	3,07	0,016

The illustrative example of pressure distribution is shown in Fig.3.

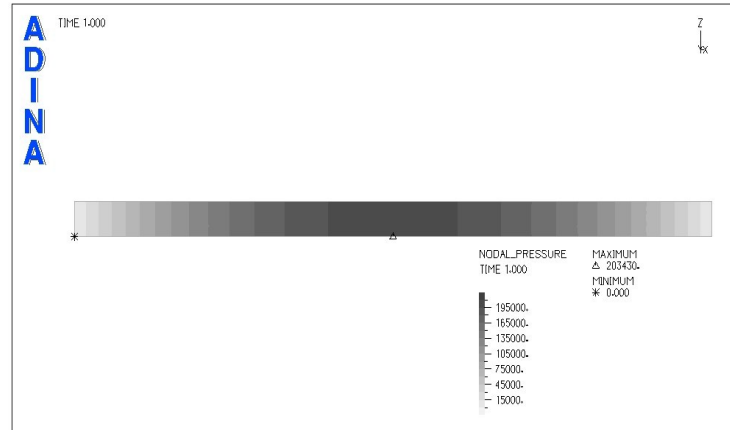


Fig. 3. Pressure distribution in the synovial film for $t=0.001$ s.

Results shown that the approach time of the plates considerably affects the pressure in the joint and the flow rate of synovial fluid [3].

5. Conclusions

In this paper the problem of human rapid movement influence on the tribological features of a synovial joint was discussed. The flow in the joint was calculated numerically using the MES solver in an ADINA-F package. To dynamically describe the synovial fluid behaviour, the nonstationary Navier Stokes equations were used. The results show that rapid human movement results in increasing the tribological quantities of the synovial joint.

The obtained results show that rapid human movement results in increasing the tribological quantities of the synovial joint.

The calculations performed pointed out great effectiveness of the ADINA-F solver in such calculations. The use of the nonstandard mesh generator in ADINA enabled choosing the optimum mesh and the time step. The number of elements which is required for a given accuracy depends on the flow geometry.

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